A Behavioral Gompertz Model for Cohort Fertility Schedules in Low- and Moderate-Fertility Populations

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Abstract

In this paper, I reintroduce the Gompertz model of age-specific fertility. Past authors have rejected this model because it fits poorly to cross-sectional, or period, rates. However, I find that the model fits very well to recent medium- and low-fertility *cohort* (rather than period) schedules in France, Italy, and Japan. A distinct advantage of the Gompertz model is that it offers a simple behavioral interpretation: that in a cohort, social diffusion of fertility behavior competes with the fertility-depressing effects of older age.

The Gompertz model plus refinements, which include better specification of the biological limits of childbearing, offers a means for forecasting future fertility, describing temporal change in fertility, and assessing the fertility-limiting effects of older entry into motherhood. In addition, this model allows for traditional uses of model age-schedules such as smoothing and correction of data.

The model estimates the completed cohort fertility of French, Japanese, and Italian cohorts born in 1965 to be 2.0, 1.6, and 1.6, respectively. For France, this number represents only a minor decline from earlier cohorts, but for Japan, the decline in cohort fertility is marked. In Italy, the Gompertz model plus a biological infertility factor suggests that the recent decline fertility in Italy results mainly from shifts to older ages of childbearing rather than from other causes.

Introduction

Low fertility is becoming a world-wide phenomenon. Nearly half the world's population exhibits period Total Fertility Rates of less than 2.1 (Wilson, 2006). Some

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of this decline stems from real declines in completed family size, while another part of it results from the "distorting" effects of postponing fertility. Period measures are good indicators of the intensity of births at a particular time, but they are poor measures of completed family size and thus of underlying tendencies for long-term fertility levels in the absence of tempo effects. While much demographic research has been occupied with finding better period measures (e.g., Bongaarts and Feeney, 1999; Kohler and Ortega, 2002a, 20002b), this paper takes the view that more cohort analysis is needed. In particular, attention needs to be paid to understanding the timing and level of cohort fertility in the form of model fertility schedules.

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The criteria for a good model of cohort fertility are several:

(1) If possible, the model should have a behavioral interpretation, and not be simply curve-fitting. Otherwise, there is no reason to expect that an arbitrary mathematical function that fits in one time and place will fit in another. Behavioral models not only offer the promise of more universality; they also allow insight when the model does not fit. Goodness-of-fit should be adequate but should not be the only factor in model selection.

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(2) The model should offer the possibility of forecasting the future fertility of incompletely observed cohorts.

(3) The model should allow forecasting with aggregate age-specific fertility rates, because parity-specific rates are often not available.

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Past Studies

A large literature exists on model fertility schedules, and most of it has focused on period schedules. Hoem et al. (1981) rejected the Gompertz model because it did not fit well to period schedules. Others have found mixed success using the Gompertz

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model (e.g., Wunsch, 1966; Valkovics and Pollard, 1992). Recent studies that have used model fertility schedules based on mixture models—two Hadwiger distributions, in the case of Chandola et al. (1999), and two normal distributions, in the case of Peristera and Kostaki (2007)—also rely on cross-sectional data. But the problem with modeling cross-sectional data is that fertility rates at different ages have no necessary relationship to each other. This lack of correlation is pronounced in recent decades, when young cohorts have been postponing births even more than older cohorts, creating cross-sectional age-distributions of fertility that have a different skewness than those of actual cohorts.[1]

The contention presented here that cohort fertility depends on social diffusion follows up on a large literature concerning the importance of social influence on individuals' fertility behavior. In recent decades, such social diffusionist explanations have become more important in explaining demographic change (National Research Council 2001). Notably, Watkins and others from the Princeton European Fertility project have argued for the dominant role of diffusion in the first demographic transition. More recently, Kohler (2000, 2001) and others have written extensively on social interaction effects in low-fertility populations. Hernes (1972) introduced social diffusion models in order to analyze cohort behavior in his study of a cohort's entry into first marriage. Goldstein and Kenney (2001) used the Hernes model to forecast first marriage in the United States.

An alternative approach to modeling fertility is to use averages of empirical schedules. Coale and Trussell (xxxx) took this approach. Their model of marital fertility—like the behavioral Gompertz model presented below—combines a biological schedule with a behavioral schedule. In this model, the biological schedule

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[1] Some older work applyies the Gompertz model to cohorts (Murphy and Nagnur, 1972; Denton and Spencer, 1974; Wunsch, 1966).

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provides the age-profile of "natural" fertility in the absence of parity-specific control, while an additional behavioral schedule gives an age-specific behavioral schedule of fertility control at older ages. As we shall see below, in the case of modern contracepting populations, the roles of behavior and biology are in some ways reversed; choice plays a role in fertility schedules at younger ages, but biology dominates the pattern at older ages.

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A Behavioral Gompertz Model

Although demographers have used the Gompertz (1825) model in the past, they seem to have done so because they were familiar with the mortality applications of the model rather than because they were exploring a specific behavioral storyline. As Hobcraft, Menken, and Preston (1982) wrote, the Gompertz model (and other models such as the Hadwiger) formed the basis of "many attempts to fit various mathematical curves, *without a behavioral or theoretical interpretation*" (p. 13, emphasis added). Similarly, Page (1997) notes, "Despite their great utility, however, models based simply on finding the function that best fits the data are not very satisfying – unless, that is, the function's parameters can be identified with biological or social processes that govern fertility" (p. 85). With these observations in mind, I present a behavioral and biological basis for the Gompertz model for fertility that is analogous to Hernes' (1972) social diffusion model of first marriage.[2]

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[2] Intriguingly, Hernes notes that he considered the Gompertz model for modeling first marriage. As he notes in his footnote 6, p. 181. "A model generating a logistic curve, for example, systematically gives a much worse fit, and hence can be eliminated. But one could make assumptions generating a Gompertz or doubly exponential curve.[Footnote 6: I first fitted a Gompertz curve directly to the cumulative percentages of first marriages; the model in (10)[the Hernes model] was developed later." Not clear here which parts of quote are from Hernes' main text and which are from a footnote.

Two points are of interest: first, that despite its form of deductive theoretical presentation, the validity of the model for Hernes rested in its goodness-of-fit. Second, the Gompertz model would seem *a priori* inappropriate for a social diffusion model of a single-decrement event such as marriage, because that model does not take into account the shrinking exposure to risk. This accounting for age-

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Let the total cumulative fertility of a cohort at age be denoted $F(x)$. (Total cumulative fertility equals the average number of births to cohort members by age .) Let $f(x) = F'(x)$ be the density of births at age (where is the age derivative of).

The simplest form of social diffusion (Coleman, 1964: 42) is proportionality, in which the rate at which individuals adopt a behavior is proportional to the number of individuals already practicing it. In the case of fertility, this form of diffusion translates to letting the fertility rate be proportional to the total cumulative fertility. Under this assumption, the more children born to a cohort, the more social pressure impels everyone in the cohort to have children. To formalize this, one can write

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where the parameter A can be understood as similar to the contagiousness of a disease. When A is larger, social pressure is more effective. The equation above is a differential equation whose solution is the exponential function: $F(x) = \exp(Ax)$ or, equivalently, $f(x) = A \exp(Ax)$. In this form, we see that such a model is clearly inappropriate for the schedule of fertility at all ages, which does not increase without bound.

A more realistic form of social diffusion includes a time—or, equivalently for a cohort, an "age"—effect. The model then has the form

In the case of fertility, it is reasonable to posit that the function $A(x)$ is declining. This decline happens for a number of reasons. Most well known is an increase in secondary sterility with age. However, social factors are also at play. For

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related factors constitutes the essential difference between the Hernes model, which has $P' = P(1-P) AB^x$, versus the Gompertz model, which has $F' = F A B^x$.

example, as people get older, they may get more set in their ways and increasingly resist the example of others, so that social pressure becomes less effective (Hernes 1972). Moreover, within a cohort, heterogeneity in desired family size—and thus in the age at which targets are reached—may occur. Such a pattern would be consistent with lower fertility at older ages. These three factors can be thought of as linked to “biological age,” “duration” of the process, and selection in a “heterogeneous population.”

Following Hernes, we let . Here the exponential is used as the simplest way to describe a continuous decline without taking negative values. This gives us the model in the form

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Solving this differential equation gives

for the cumulative fertility function. Letting and $B = \exp(-b)$, we have

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the familiar Gompertz function. Taking derivatives of the cumulative functions gives us the fertility schedules,

or equivalently, substituting for A and B,

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These functions can be fit to data using nonlinear least-squares estimates.[3] In the applications that follow, I use equation (*) to minimize the squared residuals of the fertility schedules. When trying to model fertility at late ages, it is important to model the density schedule, $f(x)$, rather than the cumulative fertility, $F(x)$, in order to give appropriate weight to observed fertility at older ages.[4]

One can interpret the parameters of the Gompertz model directly. The completed total fertility over all ages is K. As in mortality modeling, a is a location parameter, which, for a given b determines the location of the schedule along the age axis. Likewise, b is a time-scale parameter determining the rapidity of the process. A large b indicates a compressed distribution with little variance, as the cohort moves through its life relatively quickly. A small b indicates a slower time-scale of the process.[5]

The three-parameter model is not infinitely flexible. Indeed, as Volkovics and Pollard have shown, the distribution has fixed skewness and peakedness.[6] Thus, the fit of the Gompertz model across a wide number of cohorts tells us something informative about the character of cohort fertility schedules. This is also a reason that the model is often inappropriate for modeling period schedules.

Data and Analysis

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[3] For example, one can use the nls() function in R. Or in Excel, one can use the “solver” function. See Harris (1998) for a tutorial.

[4] Fitting the cumulative schedule is problematic. In effect, the cumulative schedule weights younger ages much more than older ages, because the rates at younger ages contribute to cumulative fertility at all older ages, but not vice-versa. Fitting the density directly avoids this problem.

[5] The interpretations given here differ slightly from those given earlier in the literature on Gompertz models of fertility. In particular, the understanding that b influences the time scale of the process is an insight derived directly from the Gompertz mortality literature. (See Canudas-Romo, … for a recent exposition.)

[6] As pointed out by Windsor and the appendix of Murphy and Nagur, the Gompertz distribution has the interesting property that the mode of the density occurs when a 1(number one or letter l?) no italics if #1/e share of the cumulative distribution has been reached. In our case, this means that fertility rates begin to fall when cumulative fertility has reached a level of K/e. This highlights the inflexibility of the model and the rigidity of its assumptions, for few would dare to predict K, as soon as a cohort fertility had peaked.

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In order to test the suitability of the Gompertz model across a range of circumstances, I fit the Gompertz and related models to cohorts from three populations intended to contrast a range of social, cultural, political, and economic regimes. Published cohort schedules were available for France and Japan, representing populations with moderately high and nearly lowest-low fertility. These countries also differ in the rate at which childbearing occurs within marriage: nearly all births in Japan occur to married couples, but not so in France, where pro-natalist policies are strong, (pro-natalist policies are far less strong in Japan). As a third point of comparison, I analyze Italian cohorts in order to see the suitability of the Gompertz model for a Mediterranean pattern of late, low fertility.[7] (In the future, endeavors such as the Human Fertility Data Project [MPIDR] will make it possible to do more comprehensive comparisons.)

My analysis proceeds as follows. First, I look at the success of the Gompertz model as applied to complete cohorts, in order to judge its appropriateness for modeling moderate fertility populations. Second, I look at more recent, truncated cohorts and show how the Gompertz model can be improved by incorporating an additional factor to account for declines in fecundity in the third and fourth decades of life. Finally, I show that a behaviorally and biologically inspired Gompertz approach ("Gompertz-with-infertility") produces fits that are comparable with the so-far theory-less Hadwiger model, and that the Gompertz-with-infertility model appears to be more robust than the Hadwiger for forecasting truncated cohorts with a pattern of late childbearing.

The Performance of the Standard Gompertz Model

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[7] I have also fit Danish cohort data, but this analysis is not yet complete.

The fit of the Gompertz model for complete cohorts is shown in Figure 1 for French, Japanese, and Italian cohorts born in 1945. We see that the model fits all three schedules extremely well, despite the variety of fertility levels, location, and age spreads in these three populations.

The lower panels of Figure 1 show the Gompertz model fitted to the cohort of 1965, a cohort that was last observed at age 38 in France, 39 in Italy, and 35 in Japan according to these observations. In all three countries, compared to the cohort of 1945, the cohort of 1965 has a lower level of fertility, less-concentrated fertility around the mode, and a later mode. From the figures, we see that the Gompertz model seems to fit well over the peak ages of fertility, but in these more recent cohorts, it tends to overestimate fertility at ages over 35 (approximately). This tendency occurs because biological infertility increases more quickly than the exponential function assumed in the Gompertz model.

Improving the Gompertz Model By Adding an Additional Infertility Term

A better-fitting model for fertility above age 35 can be obtained by incorporating an additional term for secondary sterility. Leridon et al. (2004) suggest a linear decline in fecundity from a level of 100 percent at age 33 to 0 at age 45 as a best estimate of population-level declines. Thus, letting the function $g(x)$ take the value of 1 before age 33 and 0 after age 45, with a linear decline in between, we can model the fertility schedule as the Gompertz function plus a multiplicative sterility effect. I define the “Gompertz-with infertility-model” as

or, written out fully, substituting from equation (*),

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Figure 2 shows what happens when one applies the Gompertz-with-infertility-model, as well as the non-theory-based Hadwiger function, to the cohorts of 1965 and 1970. The 1970 cohort is shown in order to demonstrate how the models perform when fewer ages are observed. The solid black lines are the Gompertz model, the dashed black line is the Gompertz model with infertility, and the solid grey line is the Hadwiger model.

We see that the incorporating the additional infertility term into the Gompertz model improves greatly the fit at older ages. In the Italian and Japanese cohorts of 1965, the model now predicts nearly perfectly the oldest observed fertility rates. In France, the infertility term seems to overcompensate slightly. A possible cause of this unusually high late fertility in France could be strong pro-natalist policies toward third- and higher-order births.

Comparison with the Hadwiger Model

The Gompertz-with-infertility model appears to fit moderate- and low-fertility cohort data very well. Moreover, it has the advantage of offering a plausible behavioral and biological basis. Admittedly, it is difficult to distinguish between the age-related declines in the original Gompertz model and those in the extended model with infertility, since the downward pressure on fertility at older ages captured in the declining exponential term of the Gompertz model results from several factors, including declining fecundity with age. Still, I would argue that Gompertz-with-infertility model offers a firmer behavioral and biological basis than purely mathematical curves such as the Hadwiger, Gamma, and Beta, which are used only because they provide goodness-of-fit.

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Notably, the Hadwiger model does not provide better fits than the Gompertz-with-infertility model. Indeed, in Figure 2, we see that for Italian cohorts, the Hadwiger model performs poorly, much worse than the Gompertz-with-infertility model and barely better than the standard Gompertz. For Japanese cohorts, the Hadwiger model gives nearly identical estimates as the Gompertz-with-infertility model. For France, the Hadwiger model produces rates in between the standard Gompertz and the Gompertz-with-infertility rates. Overall, therefore, there seems little reason to prefer the Hadwiger model on the basis of goodness-of-fit. In particular, as we see in the Italian case, the Hadwiger model can seriously overestimate late fertility.

Sarah Herbold 8/11/09 3:31 PM
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Sarah Herbold 8/12/09 3:01 PM
Comment: (Correct? I can't see Figure 2 for some reason. In any case, the sentence should be restructured along the lines I show here.)

Figure 3 shows goodness-of-fit comparisons among the three models across a much larger number of cohorts in Denmark, France, Italy, and Japan. The measure of goodness-of-fit used is the root-mean-squared-error between the estimated and observed age-specific rates for each cohort. In one example, the value of about 0.004 for the Danish cohort of 1955 means that the average error (in root mean square terms) for each age-specific fertility rate was about 4/1000. Age-specific fertility rates are on the order of 0.1, so all of the models are performing very well, and no single model is clearly superior.

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In general, however, the Gompertz-with-infertility model fits better than the standard Gompertz. The two estimates are identical when fertility is observed only before age 33, as is the case for the 1970 cohort in Japan. By this measure, goodness-of-fit does not show either that the Hadwiger is superior to the Gompertz-with-infertility, or vice versa. The Gompertz-with-infertility model performs slightly better for the completed cohorts (before about 1960) in France and Japan, whereas in Italy, neither the Hadwiger nor the Gompertz-with-infertility is preferable. For truncated

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cohorts (after about 1960) the Hadwiger does somewhat better (except in Italy) in fitting fertility rates at the observed ages. But as we have seen, the Hadwiger model tends to overpredict fertility at older ages that are "out-of-sample."

From the more comprehensive comparison described in Figure 2, we find that both the Hadwiger model and the Gompertz-with-infertility model tend to fit better to observed fertility rates than the standard Gompertz model. However, sum-of-squared-residuals calculations are based only on observed fertility rates, and do not factor in the tendency for the Hadwiger and standard Gompertz models to overpredict fertility at yet-to-be-observed ages.

Applications of the Gompertz Model

I now show some applications of the Gompertz model of age-specific cohort fertility. First, I show forecasts of the completed cohort fertility for Italy, Japan, and France. I then show that the Gompertz-with-infertility model can be used to estimate a hypothetical unmet need for offspring who would have been born if it had not been for the "interference" caused by infertility.

Figure 4 shows the forecast completed cohort fertility for the cohorts born from 1945 to 1965 in Italy, France, and Japan. Here, the Gompertz and Gompertz-with-infertility models are used to predict the future fertility of cohorts. The forecast complete cohort fertility is calculated by adding the fertility rates for the observed ages to the fertility rates for the forecast ages. (For example, for the cohort born in 1960, for which the last observation was in 2000, I calculate the forecast complete cohort as the sum of the observed fertility rates up to age 40 and the forecast fertility rates for older ages.)

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Taking the Gompertz-with-infertility model as the preferred one, we see that a downward trend has occurred in cohort fertility in France and Japan since the mid-1950s, and in Italy over the entire period. Cohort fertility in France remains over 2.0 for those born in 1965, whereas in Italy and Japan, the same cohort fertility is about 1.6. The rate of decline in cohort fertility is about twice as fast in Japan as in Italy, which, in turn, has a rate of decline about twice as fast as France.

The dashed lines in Figure 4 show the forecast cohort Total Fertility Rate based on the standard Gompertz model. One interesting interpretation of the two models' differing results would be to view them as a measure of the unmet need for children created by the rapid decline in fecundity at older ages. This interpretation takes the unmodified Gompertz model as an estimate of the fertility that the cohort would have had without the additional age-related infecundity. "Waiting too long" appears to account for a shortfall of about 0.1 offspring in the cohort of 1965 in these three countries. We see that if it had not been for this "delay," no decline in cohort fertility would have occurred in France and Italy from 1955 to 1965. Furthermore, the declines in cohort fertility in Italy before 1955 seem to have occurred not because an increasing share of fertility was delayed to older ages, but rather because fertility declined at all ages. Finally, biological limits of childbearing appear to play a relatively small role in Japan. Even without those limits, these models suggest, cohort fertility would have declined rapidly from the cohort of approximately 1955 onwards.

Future Research

I plan to pursue further research in several directions. The first is to expand the range of fitted populations and to incorporate cohort fertility figures from the United States as well as from the lowest-low fertility populations in Eastern Europe.

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Sarah Herbold 8/11/09 3:51 PM
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Sarah Herbold 8/12/09 3:01 PM
Comment: Need or desire? "Need" makes it sound like you're saying the society needs these births in order to maintain itself. Same issue in first graph of this section.

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The second line of research is to refine further the model of sterility. In particular, it would seem that a model that did not begin and end so abruptly would be more appropriate.

A third line of research could involve testing some of the assumptions of the social diffusion framework. Does a shock in fertility when a cohort is aged *x* appear to influence its subsequent fertility older ages? This trajectory also suggests that statistical analyisis of cohort fertility as a time series—albeit a short one—could be revealing.[8]

Third, a closer comparison of the Gompertz models with other standard mathematical functions in use such as the Hadwiger, Gamma, and Beta functions, may prove productive. In addition, behavioral assumptions that might be expected to produce these distributions could be elaborated. For example, the Gamma and Hadwiger functions both result from waiting times in stochastic processes.

Finally, I will look at the variability in the model parameters across time, both in order to provide a richer description of past fertility change and to inform forecasts of future fertility.

Conclusion

The Gompertz model appears to describe accurately cohort age-schedules of fertility in low- and moderate-fertility populations. This model also appears to be useful for forecasting the future fertility of cohorts that are still young, particularly if additional account is taken of the decline of fecundity with age. If the results presented here are found to be more broadly applicable, I would suggest that such cohort forecasts, along

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[8] Ron Lee suggested such an approach to me concerning the Hernes model. Likewise, the Gompertz model can be written as a recursive equation $F(t+1) = b0+ b1\ F(t) + b2\ F(t)*\log(F(t))$. Adding an error term to the right hand side, and estimating the coefficients by regressing $F(t+1)$ on $F(t)$ could be a promising approach. Reformulation is required, however, in order to make $F(t)$ monotonically increasing. Italics necessary for variables in this fn., as elsewhere?

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